SOME PROBLEMS IN THE THEORY OF PLANE JETS OF CONDUCTING FLUID

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Using the boundary-layer equations as a basis, the author considers the propagation of plane jets of conducting fluid in a transverse magnetic field (noninductive approximation).

The propagation of plane jets of conducting fluid is considered in several studies [1–12]. In the first few studies jet flow in a nonuniform magnetic field is considered; here the field strength distribution along the jet axis was chosen in order to obtain self-similar solutions. The solution to such a problem given a constant conductivity of the medium is given in [1–3] for a free jet and in [4] for a semibounded jet; reference [5] contains a solution to the problem of a free jet allowing for the dependence of conductivity on temperature. References [6–8] attempt an exact solution to the problem of jet propagation in any magnetic field. An approximate solution to problems of this type can be obtained by using the integral method. References [9–10] contain the solution obtained by this method for a free jet propagating in a uniform magnetic field.

The last study [10] also gives a comparison of the exact solution obtained in [3] with the solution obtained by the integral method using as an example the propagation of a jet in a nonuniform magnetic field. It is shown that for scale values of the jet velocity and thickness the integral method yields almost-exact values. In this study [10], the propagation of a free jet is considered allowing for conduction anisotropy. The solution to the problem of a free jet within the asymptotic boundary layer is obtained in [1] by applying the expansion method to the small magnetic-interaction parameter. With this method, the problem of a turbulent jet is considered in terms of the Prandtl scheme. The Boussinesq formula for the turbulent-viscosity coefficient is used in [12].

This study considers the dynamic and thermal problems involved with a laminar free and semibounded jet within the asymptotic boundary layer, propagating in a magnetic field with any distribution. A system of ordinary differential equations and the integral condition are obtained from the initial partial differential equations. The solution of the derived equations is illustrated by the example of jet propagation in a uniform magnetic field. A similar solution is obtained for a turbulent free jet with the turbulent-exchange coefficient defined by the Prandtl scheme.

1. Dynamic problem of laminar free jet. In the noninductive approximation, the initial system of equations of a laminar boundary layer for an incompressible conducting fluid has the form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}u, \qquad (1.1)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1.2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}+\frac{\sigma B^2}{\rho}u^2. \qquad (1.3)$$

In the first case we consider the dynamic problem of a plane free jet propagating in a transverse magnetic field (Fig. 1a). Here the solution to Eqs. (1.1) and (1.2) should obey the boundary conditions

$$\frac{\partial u}{\partial y}=0,\ \ v=0 \ \ \text{for } y=0,\ \ u=0 \ \ \text{for } y=\pm\infty.$$

We shall seek a self-similar solution to this problem. To do this we use the following form of the solution:

$$u=u_m F(\varphi),\ \ y=\delta(x)\varphi. \qquad (1.4)$$

Here $u_m = u_m(x)$ is the maximum velocity and $\delta = \delta(x)$ is proportional to the relative jet width.

Substituting the expressions for the velocity components u and v and their derivatives into equation of motion (1.1), we obtain, after some transformations,

$$-FF''+\frac{u_m'\delta}{u_m\delta'}(F'^2-FF'')=$$
$$=\frac{1}{u_m\delta\delta'}F'''-\frac{\sigma B^2}{\eta}\frac{\delta}{u_m\delta'}F' \qquad (1.5)$$

with boundary conditions

$$F=0,\ \ F'=1,\ \ F''=0 \ \ \text{for } \varphi=0,$$
$$F'=0 \ \ \text{for } \varphi=\pm\infty$$

(the primes denote differentiation of the functions $F(\varphi)$ with respect to φ and of the functions $u_m(x)$ and $\delta(x)$ with respect to $x\nu$).

In order to solve Eq. (1.5) it is first necessary to find the functions $u_m(x)$ and $\delta(x)$. Reference [8] gives the relationship $u_m \sim x\delta^{-2}$ as one of the equations for finding the unknowns. In [6, 7], all three quantities in Eq. (1.5) which are functions of x (arbitrarily $q_i(x)$, i = 1, 2, 3) are sought in the form

$$q_i=\text{const}+p_i(x)$$

where the functions $p_i(x)$ are assumed to be proportional. Moreover, Eq. (1.5) is divided into two independent equations which are integrated. In our opinion, such methods are artificial.

Nevertheless, to find the functions $u_m(x)$ and $\delta(x)$ we can obtain equations directly from the initial system. To do this we integrate Eq. (1.5) over the jet cross section (from $-\infty$ to $+\infty$). Here we obtain

$$1+2\frac{u_m'\delta}{u_m\delta'}+\lambda N\frac{\delta}{u_m\delta'}=0,$$
$$N=\frac{\sigma B^2}{\eta},\quad \lambda=\frac{F(\infty)}{\alpha},\quad \alpha=\int_0^\infty F'^2(\varphi)d\varphi. \qquad (1.6)$$

Allowing for (1.6), we rewrite Eq. (1.5) as

$$\frac{1}{u_m\delta\delta'}F'''+\frac{1}{2}(FF''+F'^2)=$$
$$=N\frac{\delta}{u_m\delta'}\Big[F'-\frac{\lambda}{2}(F'^2-FF'')\Big]. \qquad (1.7)$$

Separation of variables in this equation can be done for $N\delta/u_m\delta' = \text{const}$. This condition can be realized by profiling the external magnetic fields [1–3]. In order to obtain the self-similar solution we should assume the coefficient of F''' is constant for any magnetic field distribution. In particular, we can set

$$u_m\delta\delta'=4. \qquad (1.8)$$

Here, Eq. (1.7) becomes

$$F''' + 2(FF')' = 4N\frac{\delta}{u_m\delta'}\left[F' - \frac{\lambda}{2}(F'^2 - FF'')\right]. \quad (1.9)$$

As was noted in [2], when the factor in front of the brackets is constant, the expression for F' satisfies this equation for pure hydrodynamic flow:

$$F' = \mathrm{sch}^2\varphi. \quad (1.10)$$

We can verify that this expression is also a solution of Eq. (1.9) for any magnetic-field distribution.

The maximum value for the jet width and velocity is determined by Eqs. (1.6) and (1.8); these equations are written as

$$2\delta\delta'' + \delta'^2 - {}^3/_{16}N\delta\delta' = 0,\ u_m = 4/\delta\delta'. \quad (1.11)$$

In the general case of any magnetic-field distribution (N = N(x)), the given system of equations does not yield to integration in quadratures. However, this can be realized for a uniform magnetic field. We shall therefore restrict ourselves to the solution of this problem for constant N.

Integrating Eq. (1.11) for the boundary condition δ(0) = = 0 we obtain the following expressions for the unknowns:

$$\sqrt{C}x\nu = \int_0^\delta \sqrt{\delta}\exp\left(-\frac{3N}{32}\delta^2\right)d\delta,$$

$$u_m = \frac{4}{\sqrt{C}}\frac{1}{\sqrt{\delta}}\exp\left(-\frac{3N}{32}\delta^2\right). \quad (1.12)$$

In order to find the integration constant C we must obtain the integral condition. To do this we integrate Eq. (1.1), first over the jet cross section and then in the longitudinal direction. We then obtain the sought conservation condition

$$\int_{-\infty}^{\infty} u^2dy + \int_0^x\left(N\int_{-\infty}^{\infty} udy\right)dx = I_0$$

$$\left(I_0 = \int_{-\infty}^{\infty} u^2dy \quad \text{for } x = 0\right), \quad (1.13)$$

where I_0 is the value of the momentum at the jet source. As is usual in the theory of jet sources, the initial value for the momentum is assumed to be known.

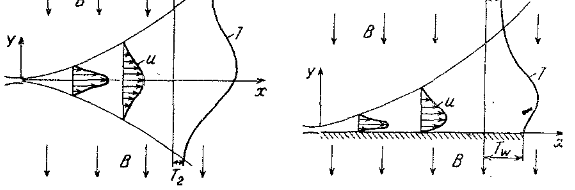

Fig. 1

With the above solution, the integral conservation condition (1.13) yields

$$\frac{64}{3C}\exp\left(-\frac{3N}{32}\delta^2\right) + 8N\int_0^\delta\frac{d\delta}{\delta'^2} = I_0. \quad (1.14)$$

By evaluating the integral we find that C = $64/3I_0$.

Thus, relationship (1.13) for the propagation of a jet of conducting fluid in a magnetic field will be an integral condition, similar to the condition for the conservation of momentum of a pure hydrodynamic jet. In the limiting case, as x → 0, expression (2.13) determines the initial jet momentum, while as N → 0 this expression becomes the momentum conservation condition

$$\int_{-\infty}^{\infty} u^2dy = I_0 = \text{const}.$$

In [7, 8] it was this limit relationship in particular that was used to determine the constant of integration

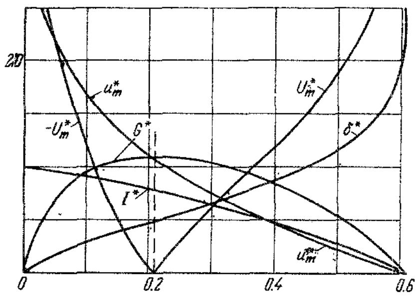

Fig. 2

for the propagation of a jet of conducting fluid; this relationship, however, is valid only for ordinary hydrodynamic flow. In [6], the constant of integration, in general, could not be found.

The conservation condition (1.13) exists when the magnetic-field strength at some point along the jet flow (for x = 0) is finite. In the case in which the external magnetic field does not satisfy this condition we can find other integral conditions allowing us to obtain a final solution. For example, when seeking solutions in powers of the magnetic field B ~ $x^{-\alpha}$ ($\alpha > 0$) we can use an integral condition of form [1]

$$\int_{-\infty}^{\infty} u^\varepsilon dy = \text{const},$$

where the constant ε is chosen so that the value of the integral is constant throughout the region of jet propagation. That the derived integral condition does not follow from the differential equations of motion is a consequence of the self-similar power transformations. In contrast, generalized integral condition (1.13) does follow from the initial differential equations and is therefore an existence condition for a nontrivial solution which is applicable for any form of solution with an everywhere-finite magnetic field strength.

We shall analyze the obtained solution. For this purpose we rewrite the expressions for the velocity components, jet thickness, and the integral quantities in a form independent of the parameter N:

$$x^* = \int_0^{\delta^*}\sqrt{\delta^*}\exp(-\delta^{*2})d\delta^*, \qquad u_m^* = \frac{1}{\sqrt{\delta^*}}\exp(-\delta^{*2}),$$

$$I^* = \frac{4}{3}\frac{I}{I_0}, \qquad v^*(\infty) = \frac{(2\delta^*)^2 - 1}{2\delta^*}, \qquad G^* = mnG,$$

$$u_m^* = mu_m, \qquad \delta^* = n\delta, \qquad x^* = px,$$

$$m = \frac{4}{\sqrt{3I_0}}\left(\frac{2}{3N}\right)^{1/4}, \qquad n = \frac{1}{4}\sqrt{\frac{3N}{2}},$$

$$p = \left(\frac{3N}{2}\right)^{5/4}\sqrt{\frac{\nu^2}{3I_0}},$$

$$\psi^* = mn\psi, \qquad u = \partial\psi/\partial y. \quad (1.15)$$

Figure 2 shows the change in these quantities along the jet. The effect of body forces leads to an increase in the relative width of the jet σ and to a more rapid decrease in velocity u_m in comparison with ordinary flow. Jet momentum I decreases along the x-axis. As the distance from the source increases, the jet flow rate

$$G = \int_{-\infty}^{\infty} u dy$$

passes through an extremum for $x \approx 0.21p$ corresponding to $\delta = 1/2$.

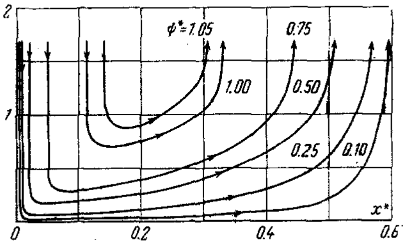

Fig. 3

At this point, the transverse velocity component z also changes sign. Thus, a decrease in flow rate is associated with fluid flow into the environment. Figure 3 shows the pattern of the streamline ψ^*. For $N \neq 0$ jet growth terminates at some finite distance from the jet source $x \approx 0.603p$. In our opinion, this physically unrealizable result is associated with the continuation of the obtained solutions into a region in which flow cannot be described by the boundary-layer equations. In fact, for small flow rates and large jet thicknesses the transverse velocity becomes commensurate with the longitudinal component. Therefore, the initial boundary-layer equations constructed for small v/u are no longer applicable.

Thus, the obtained solution to the problem of a free jet propagating in a magnetic field is applicable everywhere except the jet mouth ($x \approx 0$) and regions distant from it. The latter is due to the effects of the magnetic field on flow. These features of the obtained solution are similar to the properties of the Blasius solution for uniform flow around a finite plate; here, the solution is applicable everywhere except at the leading and trailing edges of the plate.

2. Thermal problem of free laminar jets. Consider the thermal problem for two types of boundary conditions: a) symmetric; and b) asymmetric.

a) To solve the problem for symmetric boundary conditions

$$\partial T/\partial y = 0 \quad \text{for } y = 0,$$
$$T = T_\infty \quad \text{for } y = \pm\infty \tag{2.1}$$

we assume

$$\frac{\Delta T}{\Delta T_m} = \frac{T - T_\infty}{T_m - T_\infty} = \theta(\varphi). \tag{2.2}$$

In order to find the function $\theta(\varphi)$ we convert the heat propagation equation (1.3) to an ordinary differential equation:

$$\frac{1}{4P}\theta'' + \left(1 + \frac{u_m'\delta}{u_m\delta'}\right)F\theta' - $$
$$- \frac{\Delta T_m'\delta}{\Delta T_m\delta'}F'\theta + \frac{Nu_m}{\Delta T_m\delta'}F'^2 = 0 \tag{2.3}$$

with boundary conditions

$$\theta = 1, \quad \theta' = 0 \quad \text{for } \varphi = 0, \qquad \theta = 0 \quad \text{for } \varphi = \pm\infty.$$

It is clear from Eq. (2.1) that in this case there is no self-similar temperature distribution. The temperature distribution in each section can only be obtained by numerically integrating (2.3) for corresponding u_m, δ, and ΔT.

To find ΔT_m, assuming that the jet is "hotter" than the environment by an amount

$$Q = \int_{-\infty}^{\infty} c_p u \Delta T dy \quad \text{for } x = 0$$

we obtain the following integral equation from the temperature equation (1.3) allowing for the boundary conditions

$$u_m\delta\Delta T_m = \frac{\alpha}{\beta}\int_0^x N u_m^2\delta\, dx + \frac{Q}{\beta}, \quad \beta = \int_0^\infty F'\theta d\varphi.$$

From the expression for the excess temperature ΔT_m at the jet axis obtained from (2.4),

$$\Delta T_m = \frac{Q}{\beta u_m\delta} + \frac{\alpha}{\beta}\left(\int_0^x N u_m^2\delta dx\right) u_m^{-1}\delta^{-1}. \tag{2.5}$$

It is clear that Joule dissipation leads to jet heating; this increases with an increase in the magnetic-interaction parameter.

b) For asymmetric conditions

$$T = T_2 \quad \text{for } y = -\infty,$$
$$T = T_1 \quad \text{for } y = +\infty. \tag{2.6}$$

In order to determine the dimensionless excess temperature profile

$$\frac{\Delta T}{\Delta T_1} = \frac{T - T_2}{T_1 - T_2} = \theta(\varphi) \tag{2.7}$$

we obtain the equation

$$\frac{1}{4P}\theta'' + \left(1 + \frac{u_m'\delta}{u_m\delta'}\right)F\theta' + N\frac{u_m\delta}{\Delta T_1\delta'}F'^2 = 0 \tag{2.8}$$

with boundary conditions

$$\theta = 1 \quad \text{for } \varphi = +\infty, \qquad \theta = 0 \quad \text{for } \varphi = -\infty, \tag{2.9}$$

whose solution has the form

$$\theta(\varphi) = [1 + N\theta_2(+\infty)]\frac{\theta_1(\varphi)}{\theta_1(+\infty)} - N\theta_2(\varphi),$$

$$\theta_1(\varphi) = \int_0^\varphi \exp\left(-\omega P\int_0^\varphi F d\varphi\right)d\varphi,$$

$$\omega = 4\left(1 + \frac{u_m'\delta}{u_m\delta'}\right),$$

$$\theta_2(\varphi) = 4P\frac{u_m\delta}{\Delta T_1\delta'}\int_0^\varphi\left(\int_0^\varphi F'^2\exp\left(\omega P\int_0^\varphi F d\varphi\right)d\varphi\right)\times$$
$$\times\exp\left(-\omega P\int_0^\varphi F d\varphi\right)d\varphi. \tag{2.10}$$

The temperature distribution will also not be self-similar in this case since integration is carried out for a parametric equation in terms depending on x. Instead of a monotonic temperature distribution, characteristic of pure hydrodynamic flow, Joule dissipation in this case leads to the appearance of an extremum in the temperature profile which is close to the jet axis. Figure 4 shows the qualitative dependence of the temperature distribution on the magnetic-interaction parameter.

3. Dynamic problem of a turbulent free jet. We present a similar solution to the dynamic problem for a turbulent free jet. The turbulent viscosity coefficient is found from the so-called new Prandtl equation

$$\nu_T = \varkappa\delta u_m. \tag{3.1}$$

From the initial system

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_T\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.2}$$

we obtain the following equations for determining $u_m(x)$ and $\delta(x)$:

$$1 + 2xu_m'/u_m + \lambda Nx/u_m = 0, \quad \delta' = 4\varkappa. \tag{3.3}$$

As for pure hydrodynamic flow, the dimensionless velocity profile in a turbulent jet coincides with the velocity profile of a laminar jet (1.10).

Upon integrating (3.3) for the boundary condition $\delta(0) = 0$ and using the integral equation (1.13) we obtain

$$u_m = C/\sqrt{x} - {}^1/_3\lambda Nx, \qquad \delta = 4\varkappa x,$$

$$C = {}^1/_4\sqrt{3I_0/\varkappa}. \tag{3.4}$$

Figure 5 shows the change in maximum velocity u_m/u_0 along the jet axis, where $u_0 = c/(x)^{1/2}$ is the value of the velocity at $N = 0$.

The derived solution has one important drawback for any finite value of the parameter N: the velocity, beginning with some point, passes through zero and then becomes negative. This physically impossible result evidently indicates that the solutions in a laminar jet are not applicable at finite distances from the jet mouth. Naturally, the application of the Prandtl scheme for turbulent streamline flow in a magnetic field requires experimental verification.

4. Dynamic problem of a semibounded laminar jet. For jet propagation at the wall (Fig. 1b) the boundary conditions take the form

$$u = 0, \quad v = 0 \quad \text{for } y = 0; \; u = 0 \quad \text{for } y = +\infty.$$

By converting the self-similarity condition

$$u = u_m F(\varphi), \qquad y = \delta(x)\varphi$$

used above, we reduce Eqs. (1.1) and (1.2) to the following system of ordinary differential equations:

$$F''' + FF'' + 2F'^2 = \frac{3N\delta}{u_m\delta'}\left[F' - \frac{\xi}{2}(F'^2 - FF'')\right], \tag{4.1}$$

$$2 + 3\frac{u_m'\delta}{u_m\delta'} + N\xi\frac{\delta}{u_m\delta'} = 0, \qquad u_m\delta\delta' = 3, \tag{4.2}$$

$$\xi = \frac{F^2}{\gamma}, \qquad \gamma = \int_0^\infty FF'^2\,d\varphi. \tag{4.3}$$

For a uniform magnetic field (N = const) the solution to (4.2) and (4.3) for boundary condition $\delta(0) = 0$ has the form

$$C^{1/3}x\nu = \int_0^\delta \sqrt[3]{\delta}\exp\left(-\frac{N\xi}{18}\delta^2\right)d\delta,$$

$$u_m = \frac{3}{\sqrt{C^{2/3}}}\frac{1}{\delta^{1/3}}\exp\left(-\frac{N\xi}{18}\delta^2\right). \tag{4.4}$$

The integration constant C, defined by the integral condition

$$\int_0^\infty u^2\left(\int_0^y u\,dy\right)dy + \int_0^x N\left[\int_0^\infty u\left(\int_0^y u\,dy\right)dy\right]dx = E_0, \tag{4.5}$$

is given by

$$C = \frac{27\gamma}{E_0} \qquad \left(E_0 = \int_0^\infty u^2\left(\int_0^y u\,dy\right)dy \quad \text{for } x = 0\right).$$

Thus, as for the problem of a free jet, the equations for finding u_m and δ are only integrated for a uniform magnetic field. Here the obtained solutions for a semibounded jet have features similar to those of the solution to the problem of a free jet. Integration of the equations for finding the velocity profile (4.1) can only be carried out using the local-similarity method, i.e., by simultaneously solving (4.1)-(4.3). In this case, therefore, the velocity profiles will not be self-similar. Self-similar velocity profiles can only be obtained by choosing particular values for the external magnetic field whose form is given by the condition $3N\delta/u_m\delta'$ = const. Given this value for the induction, the field must attenuate in inverse proportion to the relative jet width $B = LB_0/\delta$. Here Eqs. (4.1)-(4.3) diverge. Numerical integration of the equation

$$F''' + FF'' + 2F'^2 = \overline{N^\circ}\left[F' - \frac{\xi}{2}(F'^2 - FF'')\right],$$

$$\overline{N^\circ} = \frac{\sigma(B_0L)^2}{\eta} \tag{4.6}$$

is performed in [4]. Equations (4.2) and (4.3) have the solutions

$$u_m = \frac{1}{C_1^2}\frac{12-N}{3}x^{-(6+N)/(12-N)}, \qquad \delta = C_1x^{9/(12-N)}. \tag{4.7}$$

In this case there is no integral condition (4.5). This is due to the distribution of the external magnetic field in the jet source: the body force becomes infinitely large for x = 0. Therefore, the integration constant C_1 is found from the integral condition used in [3, 4]:

$$C_1 = \left(\frac{3}{12-N}\right)^{9/(N-12)}\left[D\left(\int_0^\infty F'^{9/(6+N)}d\varphi\right)^{-1}\right]^{(N+6)/(N-12)},$$

$$D = \int_0^\infty u^{9/(6+N)}\,dy = \text{const}. \tag{4.8}$$

The derived relationships in (4.7) differ from the solution obtained in [4] since in the latter case the solution is sought in the form of a power series. However, the characteristics of the solution in [4] are also valid for this case. In particular, this applies to values of N limited to $0 \le N < 12$ where there are self-similar solutions: the values for N are obtained from the condition that the jet flow rate increases along the axis.

From expression (4.7) it is clear that an increase in body force leads to an increase in the relative width of the jet and a decrease in its maximum velocity.

5. Minimum problem of semibounded jet. We shall restrict ourselves to a solution to the problem of a jet propagating along a wall with a fixed temperature:

$$T = T_w \quad \text{for } y = 0, \qquad T = T_\infty \quad \text{for } y = \infty. \tag{5.1}$$

Assuming that

$$\frac{\Delta T}{\Delta T_w} = \frac{T - T_\infty}{T_w - T_\infty} = \theta(\varphi), \tag{5.2}$$

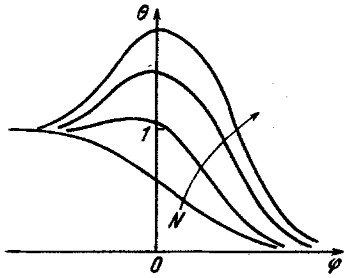

Fig. 4

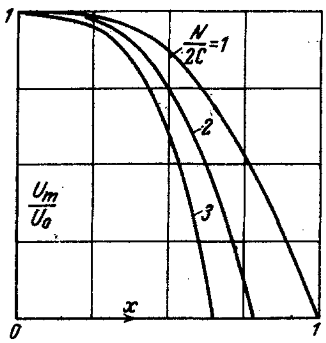

Fig. 5

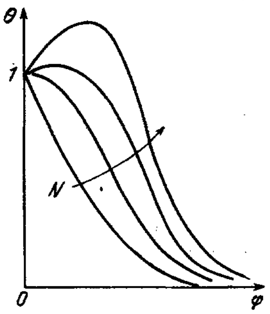

Fig. 6

we replace Eq. (2.3) by

$$\frac{1}{3P}\theta'' + \left(1 + \frac{u_m'\delta}{u_m\delta'}\right)F\theta' + N\frac{u_m\delta}{\Delta T_w\delta'}F'^2 = 0 \qquad (5.3)$$

with boundary conditions

$$\theta = 1 \quad \text{for } \varphi = 0, \qquad \theta = 0 \quad \text{for } \varphi = \infty,$$

whose solution has the form

$$\theta(\varphi) = 1 - [1 - N\theta_2(\infty)]\frac{\theta_1(\varphi)}{\theta_2(\infty)} - N\theta_2(\varphi),$$

$$\theta_1(\varphi) = \int_0^\varphi \exp\left(-\omega P\int_0^\varphi F\,d\varphi\right)d\varphi,$$

$$\omega(x) = 3\left(1 + \frac{u_m'\delta}{u_m\delta'}\right),$$

$$\theta_2(\varphi) = 3P\frac{u_m\delta}{\Delta T_w\delta'}\int_0^\varphi\left(\int_0^\varphi F'^2\exp\left(\omega P\int_0^\varphi F\,d\varphi\right)d\varphi\right)\times$$

$$\times\exp\left(-\omega P\int_0^\varphi F\,d\varphi\right)d\varphi. \qquad (5.4)$$

From the expression for the local value of the Nusselt number

$$N_x = -\frac{x}{\delta}\frac{1 - N\theta_2(\infty)}{\theta_1(\infty)},$$

$$N_x = \frac{q_x}{\lambda\Delta T_w}, \quad q = -\lambda\frac{dT}{dy}, \qquad (5.5)$$

we can estimate the thermal fluxes appearing in the jet as a result of Joule dissipation. It is clear from formula (5.5) that the thermal flux due to Joule dissipation increases with an increase in N. Here the decelerating effect of the magnetic field on jet motion leads to a more uniform temperature distribution near the wall.

Qualitatively, the temperature profiles are represented by the curves in Fig. 6.

Note in conclusion that all of the above-derived results reduce to the corresponding solutions for pure hydrodynamic flow in the absence of a magnetic field (N = 0).

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